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On the Báez-Duarte criterion for the Riemann hypothesis

Goubi Mouloud

This work is an attempt to prove the existence of a Family of Beurling functions satisfying Báez-Duarte Criterion for Riemann Hypothesis.

Let the Hilbert space $\mathcal{H} = L^2\left([0,+\infty[, t^{-2}dt\right)$ with the inner product $\langle f,g\rangle = \int_0^{+\infty} f(t)\overline{g(t)}t^{-2}dt$. For any integer n let as consider the functions e_n defined over $\mathcal{H}$ by $e_n(t) = \left\{\frac{t}{n}\right\}$. Only the Beurling functions are of the form $f_n = ce_1 + g_n$ where g_n is the sum $\sum_{k=2}^{n} c_k v_k$ and $v_k(t) = e_n(\lfloor t \rfloor)$.
Supposing that $\sum_{k\geq 0}\frac{g_n(k)}{k(k+1)}$, $\langle e_1, g_n\rangle$ and $\sum_{k\geq 0}\frac{g_n^2(k)}{k(k+1)}$ are converging respectively to α, λ and β when n tends to infinity. And using technics from Hilbert geometry, the limit when n tends to infinity of the distance of the characteristic function χ of the interval $[1,+\infty[$ to the subspace generated by f_n ([2], [3]) is

$$\lim_{n\to\infty} d_n^2(\chi, f_n) = 1 - \frac{(1-\gamma)^2c^2 + 2c(1-\gamma)\alpha + \alpha^2}{c^2(\log 2\pi - \gamma) + 2c\lambda + \beta}$$

By means of Báez-Duarte criterion [1] the Riemann hypothesis holds if

$$\left(\log 2\pi + \gamma - \gamma^2 - 1\right)c^2 + 2\left(\lambda - (1-\gamma)\alpha\right)c + \beta - \alpha^2 = 0.$$

This expression is an equation of second degree on c and it's discriminant Δ' is only negative. If $\Delta' = 0$ we get $c = \frac{(1-\gamma)\alpha-\lambda}{\log 2\pi-\gamma-(1-\gamma)^2}$.

References

[1] L. Báez-Duarte, News versions of the Nyman-Beurling criterion for the Riemann hypothesis, *Int. J. Math. Math. Sci.* Vol. 31, Issue 7, (2002), pp.387-406.

[2] A. Bayad, M. Goubi, Proof of the Möbius conjecture revisited, *Proc. Jangjeon Math. Soc.*, Vol. 16 (2013), No. 2, pp. 237-243.

[3] M. Goubi, A. Bayad and M.O. Hernane, Explicit and asymptotic formulae for Vasyunin-Cotangent sums, *Pub. Inst. Math.*, Vol. 102 (2017) 116, pp. 155-174.

Goubi Mouloud
department of Mathematics, UMMTO university,
15000 Tizi-ouzou Algeria.
email: mouloud.ummto@hotmail.fr