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Correlations of Multiplicative Functions

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Let $g_j : \mathbb{N} \to \mathbb{C}$ be multiplicative functions such that $|g_j(n)| \leq 1$ for all n. Let $F_1(x), F_2(x), F_3(x)$ are relatively co-prime polynomials.

Consider the following triple correlation function:

$$M_x(g_1, g_2, g_3) = \frac{1}{x} \sum_{n \leq x} g_1(F_1(n))g_2(F_2(n))g_3(F_3(n)). \qquad (1)$$

In [KAT], Kátai studied the asymptotic bahaviour of the above sum (1) when $F_j(x)$ are special polynomials but did not provide error term. In [ST4], Stepanauskas studied the asymptotic formula for sum (1) with explicit error term when $F_j(x), j = 1, 2, 3$ are linear polynomials.

The aim of the article [DAR] is to prove the following statement:

Theorem 1 *Let $F_j(x), j = 1, 2, 3$ be polynomials as above of degree greater than or equal to 2. Let g_1, g_2 and g_3 be multiplicative functions as above. Then there exists a positive absolute constant c and a natural number γ such that for all $x \geq r \geq \gamma$ and for all $1 - \frac{1}{v_1+v_2+v_3} < \alpha < 1$, we have*

$$M_x(g_1, g_2, g_3) - P'(x) \ll \frac{1}{x} \left(F_1(x)F_2(x)F_3(x)\right)^{1-\alpha} \exp\left(\frac{cr^\alpha}{\log r}\right)$$
$$+ (T(x))^{\frac{1}{2}} + (S(r, x))^{\frac{1}{2}} + (r \log r)^{-\frac{1}{2}} + \frac{1}{x}C(r, x) + \frac{1}{\log x}$$

where v_j denote the degree of the polynomials $F_j(n)$ respectively.

The following corollary is a direct application of the Theorem 1.

Corollary 2 *Let $\phi(n)$ be Euler's totient function and $\sigma(n) = \sum_{d|n} d$. Let $F_1(x) = x^2 + b, F_2(x) = x^2 + c, F_3(x) = x^2 + d, 0 < t < 1,$ where b, c, d are taken such that $F_j(x), j = 1, 2, 3$ satisfies the assumption of Theorem 1 and is a quadratic residue for all odd prime p. Then there exist a natural number γ such that for all $x \geq \gamma$,*

$$\frac{1}{x}\sum_{n\leq x}\frac{\phi(n^2+b)\phi(n^2+c)\phi(n^2+d)}{\sigma(n^2+b)\sigma(n^2+c)\sigma(n^2+d)} = P_1'(\gamma)\prod_{p>\gamma} w_p' + O\left(\frac{1}{(\log x)^t}\right)$$

where $w_p' = \left(1 - \frac{6}{p} + 6\left(1-\frac{1}{p}\right)^2 \sum_{m=1}^{\infty} \frac{1}{1+p+\cdots+p^m}\right).$

For more details see [DAR].

References

[DAR] P. Darbar *Triple correlations of multiplicative functions*, Acta Arithmetica, **180**, 2017, pp. 63-88.

[KAT] I. Kátai *On the distribution of Arithmetical functions*, Acta Mathematica Academiae Scientiarum Hungaricae, **20** (1-2), 1969, pp. 60-87.

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