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The Sierpiński d-dimensional tetrahedron and a Diophantine nonlinear system

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The Sierpiński tetrahedron Δ^d is the d-dimensional generalization of the most known Sierpiński gasket which appears in many fields of mathematics and applied sciences. Starting from a generating sequence of d-polytopes $\{\Delta_n^d\}_n$ for Δ^d (where Δ_0^d is the unitary d-simplex), we find closed formulas for the sum $v_n^{d,k}$ of the measures of the k-dimensional elements of Δ_n^d, deducing the behavior of the sequences $\{v_n^{d,k}\}_n$ at infinity, both in traditional analysis and in a recently proposed setting based on the symbol ①. The interesting point for us is that the use of such a new framework (just at a notational level) lead us to formulate several problems in form of Diophantine systems, which can be studied and investigated in terms of classical number theory by working with traditional tools from algebra, analysis, etc.
Complex problems arise in this way and, in particular, in the considered case we come to the following Diophantine system (see [1] for details)

$$\begin{cases} \dfrac{\sqrt{k+1}}{k!\sqrt{2^k}} \cdot \dbinom{d+1}{k+1} = \dfrac{\sqrt{h+1}}{h!\sqrt{2^h}} \cdot \dbinom{t+1}{h+1} \\[2ex] \dfrac{d+1}{2^k} = \dfrac{t+1}{2^h} \end{cases} . \tag{1}$$

Equations like the previous are not properly "Diophantine" because this word usually refers only to equations of polynomial or exponential type. Moreover, they are not very present in literature and still very little studied: just few authors call them *binomial Diophantine equations*.

The problem of deciding whether there are nontrivial integer solutions of a system like (1), and if so to find them all, is not a simple matter in general; for example, by using the most powerful scientific computational software available today (like, for instance, *Mathematica*® 11.0 or many others) it is not possible to obtain any answer except for very small values of d and t, cause the complexity of (1).

In conclusion, while if we vary the size of the starting d-simplex Δ_0^d in an appropriate way we achieve systems with nontrivial integer solutions, in our case instead, as consequence of stronger theoretical results, we obtain the following

Corollary 1 *There are no integer solutions* $(d, t, k, h) \in \mathbb{N}^4$ *of the system* (1)*, such that* $1 \leqslant k \leqslant d$*,* $1 \leqslant h \leqslant t$ *and* $2 \leqslant d < t$*.*

References

[1] Caldarola F. The exact measures of the Sierpinski d-dimensional tetrahedron in connection with a Diophantine nonlinear system, Comm Nonlinear Sci Num Simul 63 (2018), 228–238.

[2] Caldarola F. The Sierpinski curve viewed by numerical computations with infinities and infinitesimals, Appl. Math. Comput. 318 (2018), 321–328.

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